WATER BALANCE ANALYSIS OF ST. MARY LAKE, BRITISH COLUMBIA

by

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Abstract

Several key hydrological parameters were derived from data obtained between 2007 and 2014, a period when outflows were more-or-less consistently controlled by a weir in Duck Creek. A water balance model was used for this purpose. It was solved for weekly changes in lake storage, taking advantage of a new stage-discharge curve for the weir and actual data on drinking water withdrawals. Results were obtained for basic annual relationships between precipitation and total inflow, as well as discharge to Duck Creek and storm water runoff. In addition, new relationships between storm water runoff, direct rainfall to the lake and water exports have been determined on a monthly basis. These provide insight into the factors governing nutrient loading.

Useful relationships [I = total inflow, Q = outflow to Duck Creek, SW = inflow from storm water runoff in units of dam^3/yr, P = precipitation in mm/yr]

$I = 7.8\ P - 3378$

$Q = 0.89\ I - 1329$

$SW = 0.79\ I - 826$

The analysis showed that storm water runoff exceeds water removed from the lake (Duck Creek and withdrawals) for only one or two months each year, typically between November and January. In five of the eight years examined, the greatest amounts of runoff were found in November and December, when nutrient loading through internal processes is highest. In other months, outflows and withdrawals typically exceed runoff.

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[1] This report was prepared by the author to document curiosity-driven research. The views expressed herein are those of the author and not of any other person or agency.

1.0 INTRODUCTION

A water balance model has been derived for St. Mary Lake (Fig. 1.1), British Columbia, to provide estimates of the timing and magnitude of inflows to the lake, and more specifically, the seasonal variations in storm water runoff in relation to water exports from the lake due to outflow into Duck Creek and licensed water withdrawals. This is motivated by the need to better understand external nutrient loads to the lake, and the balance between these inputs and external sinks that permanently remove nutrients.

It is feasible to make this type of calculation because the data resources for the lake are reasonably comprehensive. Specifically, data for lake level, air temperature, rainfall and water withdrawals are known with suitable precision and sampling frequency.

Watershed area	645 ha
Lake surface area:	182 ha
Lake volume	16,600 dam^3

Figure 1.1 Aerial photograph of St. Mary Lake.

2.0 WATER BALANCE MODEL

2.1 Mathematical Formulation

The terms used in the water balance model are defined on the schematic shown in Fig. 2.1. The model equates the change in storage to the sum of the fluxes of water flowing into the lake, and out of the lake, and is given by:

$$A\,dh/dt = I - (\,Q(h) + W + E + GW\,) \qquad (1)$$

where A = surface area of the lake (assumed constant over the range of change in h)
h = water level
t = time
I = total inflow
Q = volume out Duck Creek, which is a function of water level
W = withdrawal by all license holders
E = evaporation
GW = loss/gain to or from ground water.

It is convenient to think of the fluxes in units of dam^3/day[2].

In standard form (1) becomes:

$$dh/dt + q(h) + (I - (W + E + GW\,))/\,A = 0 \qquad (2)$$

where q = Q/A.

Equation (2) is solved for I using the following finite difference equation:

$$I^{n+1} = A \times \Delta h/\Delta t + Q(h^n) + (W + E + GW\,) \qquad (3)$$

where n is a time step index and $\Delta h = (h^{n+1} - h^n)$.

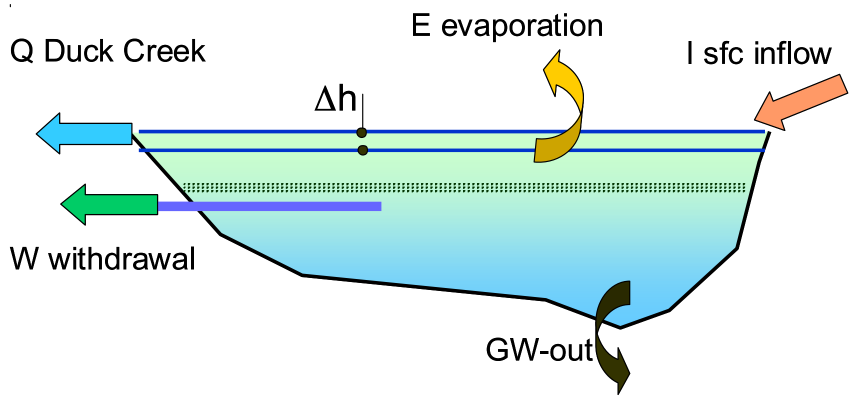


Figure 2.1 Schematic representation of St. Mary Lake and definition of nomenclature.

[2] Units: it is convenient in hydrology to use volume units of cubic decametres (dam^3) because it yields values ranging from order 10 to 10,000. 1 dam^3 = 1,000 m^3.

2.2 Definition of Terms

(i) Discharge to Duck Creek Q

St. Mary Lake has only one tributary that drains water from the lake, Duck Creek, located at its southernmost end. Since the beginning of 2007 flows have been controlled by a weir, for which a stage-discharge curve has been developed. A standard form has been assumed for this curve, $Q = ch^b$. The measurements and the fit of the curve to the data are shown in Fig. 2.2. The relationship used in the water balance is:

$$Q = \exp(432.1 \ln(h) – 1604), \text{ for } h > 40.7 \text{ m} \qquad (4)$$

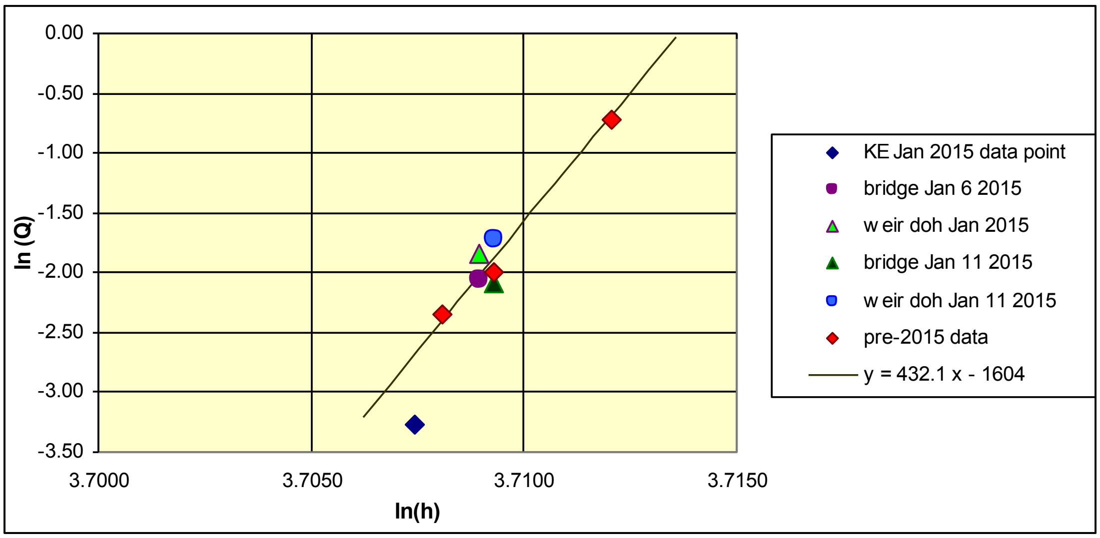


Figure 2.2 Stage-discharge curve for Duck Creek weir (after 2007).

For h ≤ 40.7 m, a constant flow of 0.009 m^3/s is specified. This is a fish conservation flow requirement for operation of the weir.

There is a fair degree of uncertainty in estimates of Q from (4). First, there is the measurement error associated with determining flow in an irregular creek cross-section, which is difficult to estimate. Most of the measurement points in Fig. 2.2 were obtained at a section just downstream of the weir structure itself. This was done to capture all of the flow passing over the weir and through the fish ladder. Second, the weir cross section was sometimes modified during winter by beavers, and periodically their debris was removed. Such variations are not accounted for in the stage-discharge curve.

Third, measurements of h used to develop the curve in (4) were made at the weir and are probably accurate to within 0.005 m. However, the measurements of h in the long-term record were made at the water treatment facility, about half-way up the lake from Duck Creek. A conversion factor was determined by correlating near-simultaneous readings, yielding a value of 40.070 m. This value has subsequently been verified by a land survey in 2015.

(ii) Withdrawals W

Withdrawals are principally to supply drinking water. The agencies responsible for the largest withdrawals are the North Salt Spring Waterworks District (NSSWD) and the Capital Regional District (CRD). Daily records, with the odd gap, were available for the NSSWD withdrawals from 2007 onward

(Fig. 2.2) and were used directly in (3) for the estimate of W. Monthly data from 2009 were obtained from the CRD. These were averaged over five years, combined with estimates of the other license holders, and extrapolated to daily values for solution of (3). This procedure yielded a realistic monthly variation over a year, but without information on daily fluctuations.

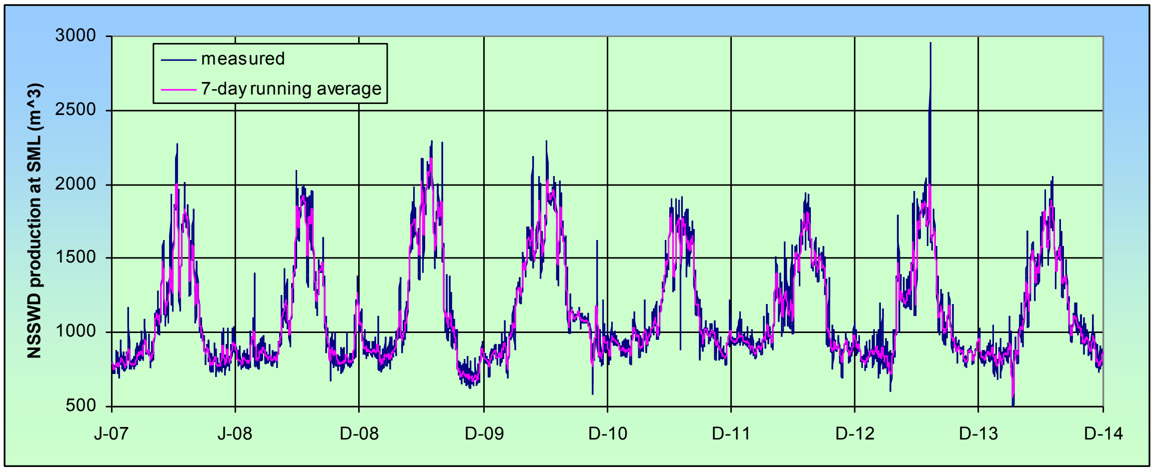


Figure 2.3 Daily withdrawal by the NSSWD. A 7-d moving average is also shown.

As of 2014 typical annual withdrawal volumes were about 400 dam^3 by NSSWD, 105 dam^3 by the CRD and, roughly, 83 dam^3 by all other license holders.

(iii) Evaporation E

The only data record available at St. Mary Lake to estimate evaporation was daily air temperature. Accordingly, the Thornthwaite equation[3] for evaporation was used, with monthly average air temperature for input. This yielded monthly values for evaporation, which were extrapolated to equal daily values for solution of (3), and applied uniformly to all years[4]. The predicted annual total evaporation is 656 mm, which is significant compared with total rainfall in the range of 700-1400 mm.

Other studies have used Environment Canada's record of evaporation at Saanichton as a benchmark for comparison. The Thornthwaite-derived values for St. Mary Lake are compared with this record in Table 2.1. The agreement is good and well within the uncertainty associated with the highly simplified Thornthwaite model. It is interesting to note from this table that the net evaporation from May through September is about 308 mm, or 1.0 feet, and confirms the 'rule of thumb' of about a foot of evaporation each year for southern Vancouver Island.

Table 2.1 Comparison of predicted and measured potential evapotranspiration.

[3] See e.g., Xu, C-Y and V.P. Singh, 2001. Evaluation and generalization of temperature-based methods for calculating evaporation. *Hydrol. Process*. **15**, 305–319 (2001) for a discussion of the Thornthwaite equation.

[4] In principle, evaporation can be calculated for actual monthly temperature data, and thus varied for each year of simulation. Such an improvement is readily incorporated into the water balance model.

	Saanich-Victoria	Saanichton CDA	Saanichton CDA	St. Mary Lake	Calculated	St. Mary Lake
	1951-80	1960-90		1976-2013	Thornthwaite	
	Precipitation	Mean Evaporation	Net Evaporation Loss	Precipitation	Mean Evaporation	Net Evaporation Loss
	(mm)	(mm)	(mm)	(mm)	(mm)	(mm)
Jan	164.7	33.4	-131.3	148.0	11.3	-136.7
Feb	105.5	8.3	-97.2	99.4	15.1	-84.4
Mar	73.1	29.5	-43.6	92.0	28.3	-63.6
Apr	42.4	57.5	15.1	54.4	47.2	-7.2
May	26.0	86.9	60.9	44.9	77.4	32.5
Jun	25.5	99.5	74.0	36.8	98.9	62.1
Jul	17.5	114.0	96.5	21.3	118.9	97.7
Aug	25.5	92.5	67.0	28.3	109.7	81.3
Sep	40.4	61.1	20.7	40.8	75.3	34.5
Oct	87.1	30.5	-56.6	89.4	43.2	-46.2
Nov	137.3	13.9	-123.4	160.9	19.8	-141.2
Dec	172.5	10.1	-162.4	151.0	11.1	-139.9
Total	917.5	637.2	-280.3	967.2	656.2	-311.0

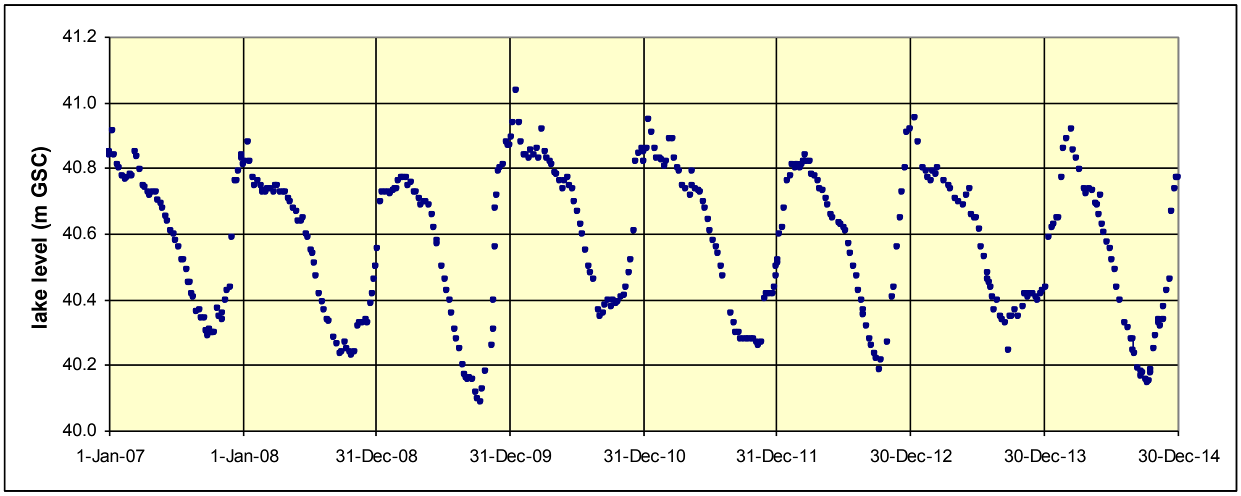
(iv) Ground water GW

There is no reliable information on shallow ground water exchange with St. Mary Lake. For the present calculation, GW was subsumed into I. It is usually considered a small percentage of the surface inflow. For example, Nordin et al. (1983[5]) assumed it would contribute (add to the lake) 5% of the total inflow.

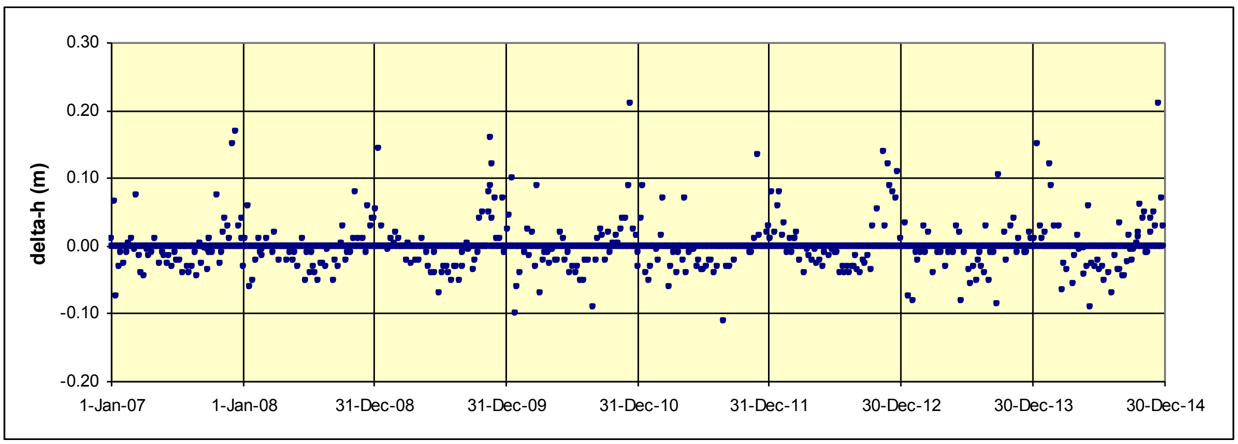
2.3 Solution of Equation (3)

Water level data were recorded weekly between 2007 and 2014. As a result, a time step Δt of approximately one week was used to step forward the calculation for I in (3). It is not quite 7 days every step; there are variations of perhaps a day or two around the one week interval due to the timing of reading the gauge. The water level time-series is shown in Fig. 2.4 together with the derived values for Δh. Water level recorded at the treatment facility was converted to elevation using the factor 40.070.

[5] Nordin, R.N., C.J.P. McKean and J.H. Wiens, 1983. St. Mary Lake Water Quality: 1979-1981. Working Report, BC Min. of Environ., File 64.080302.



(a)



(b)

Figure 2.4 (a) Measured water level (converted to m CGVD28 using the factor 40.070) and (b) calculated Δh.

This sampling frequency is not ideal. There were certainly intense rain events that occurred between sampling times, which produced water level fluctuations that were not observed and which accordingly are not reflected in the estimated inflow. Examination of the daily precipitation record suggests these events were not frequent, and do not greatly influence the estimated overall water balance as a result.

2.4 Results

The solution to (3) is shown in Fig. 2.5 with the approximately weekly time step. Figure 2.6 shows these results summed to monthly values.

The annual inflow values, calculated for a hydrological year spanning August 1st to July 31st, are shown in Table 2.2.

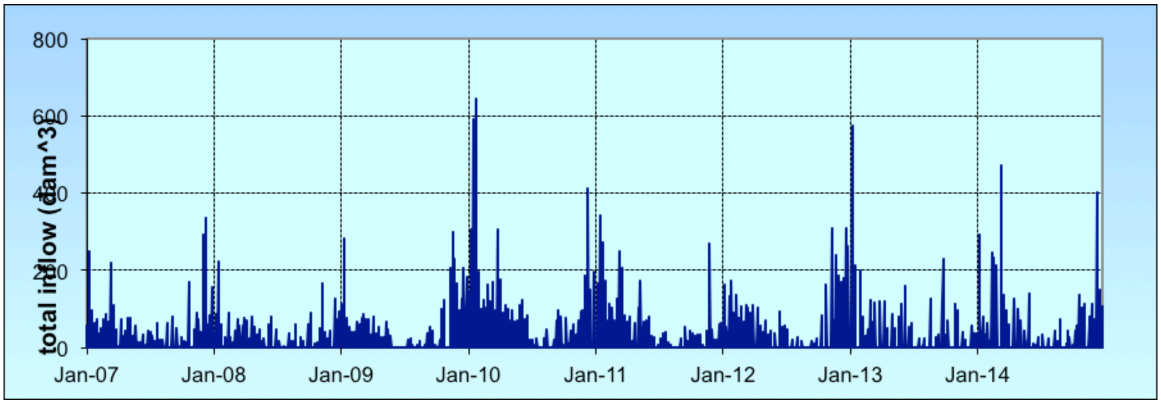


Figure 2.5 Solution to equation (3) for total inflow I in dam^3.

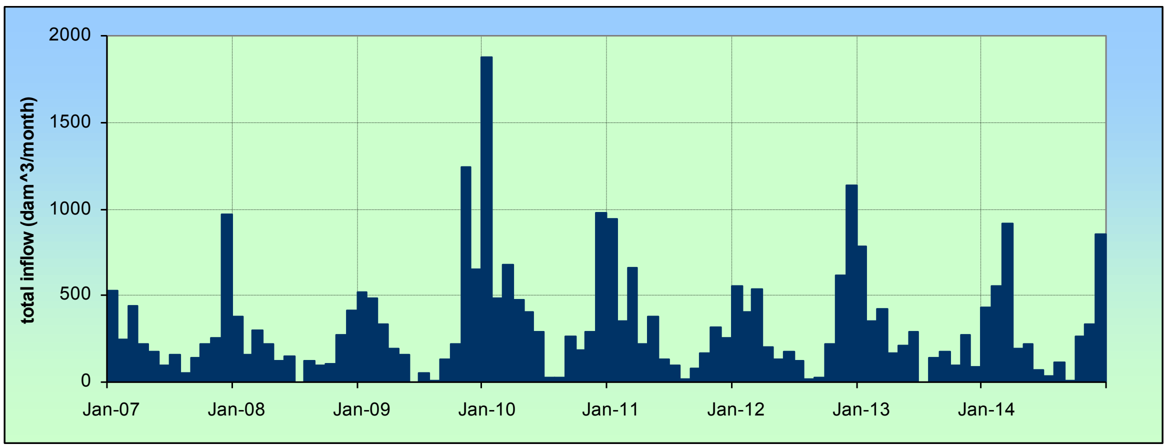


Figure 2.6 Total inflow I in dam^3/mo, calculated from the results shown in Fig. 2.5.

Table 2.2 Total inflow and total precipitation for hydrological years spanning August 1 – July 31.

Winter of	Total sfc Inflow (dam^3)	Inflow Factor	Total Precipitation (mm)
2007-08	2,958	55%	832
2008-09	2,727	58%	733
2009-10	6,508	85%	1,194
2010-11	4,529	65%	1,078
2011-12	2,972	54%	860
2012-13	4,239	64%	1,024
2013-14	3,198	61%	824

3.0 HYDROLOGICAL ANALYSIS

3.1 Inflow-Precipitation Relationship

The data in Table 2.2 are shown in Fig. 3.1, together with the relation presented in Nordin et al. (1983). The regression line is:

$$I = 7.8\ P - 3378 \qquad (5)$$

where P = precipitation in mm. The R^2 is 0.91.

The values derived in the present analysis are significantly greater than the earlier results. It is noted that Nordin et al. essentially extrapolated the curve derived for Cusheon Lake to St. Mary Lake, retaining the same slope and estimating that the precipitation yielding no runoff as 500 mm. The intercept for the regression in (5) is about 433 mm.

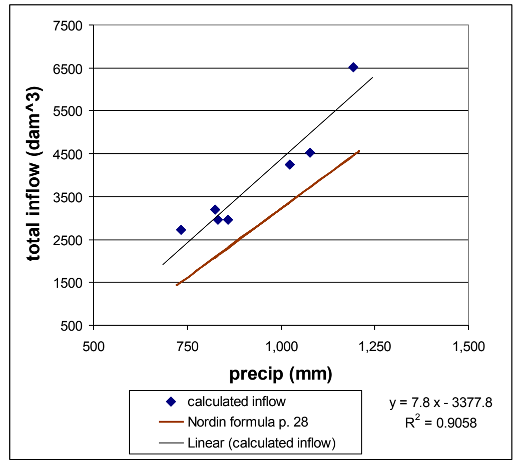


Figure 3.1 Inflow-precipitation relation for St. Mary Lake.

3.2 Flushing Times

Flushing rate, in yr^{-1}, is defined by the ratio of the total inflow to the total lake volume. The water retention time is given as the inverse of flushing time. The results are listed in Table 3.1 and plotted in Fig. 3.2. The means and ranges are: flushing rate 0.23 (0.17, 0.39) yr^{-1}, and retention time 4.6 (2.5, 6.1) yr for n=7. As expected, given the large range in inflow values there is a considerable range in water retention times.

Table 3.1 Water retention times and flushing rates for St. Mary Lake.

	Total inflow	Water retention time	Flushing rate
winter of	(dam^3)	(yr)	(1/yr)
2007-08	2,958	5.6	0.18
2008-09	2,727	6.1	0.17
2009-10	6,508	2.5	0.39
2010-11	4,529	3.6	0.27
2011-12	2,972	5.6	0.18
2012-13	4,239	3.9	0.26
2013-14	3,198	5.2	0.19
mean		4.6	0.23

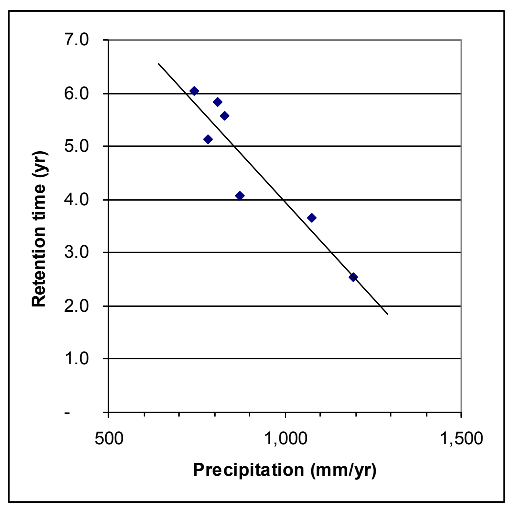


Figure 3.2 Retention time vs precipitation for St. Mary Lake.

3.3 Duck Creek Discharge

Figure 3.3 shows the weekly variations of the calculated discharge out Duck Creek. These values are summed to give monthly values (Fig. 3.4), and the annual totals (hydrological year) are shown in Table 3.2. As expected there is a large variation in the volume discharged into Duck Creek, related to precipitation and hence inflow to the lake. As shown in Fig. 3.6, a strong linear relationship between Q and I is evident. In a very dry year, as little as 30% of the inflow is discharged to the creek. In wet years, this proportion increases to more than 60%.

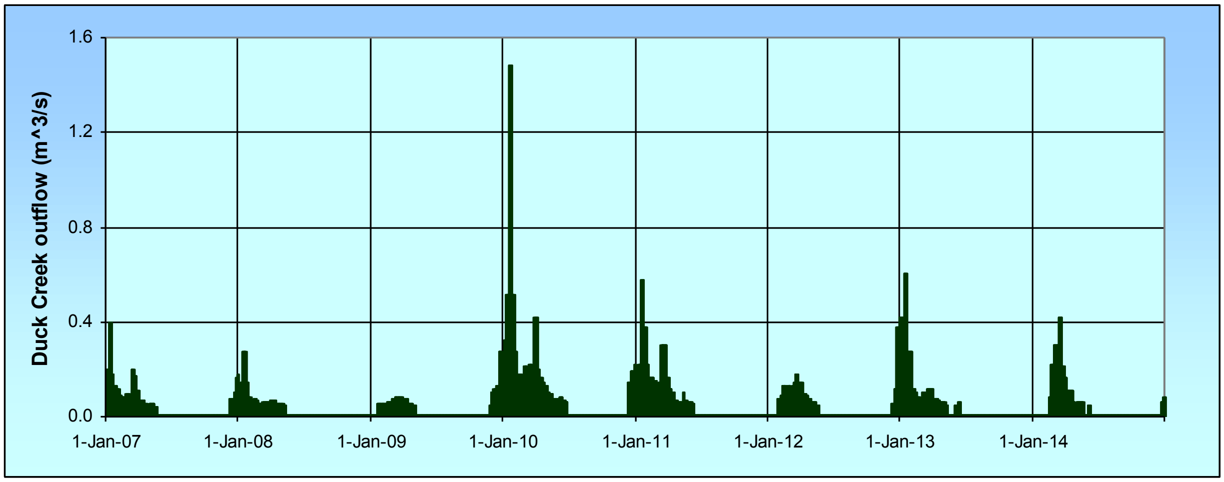


Figure 3.3 Time-series of estimated flow out Duck Creek Q in m^3/s. Conversion factor for water level is 40.070 m.

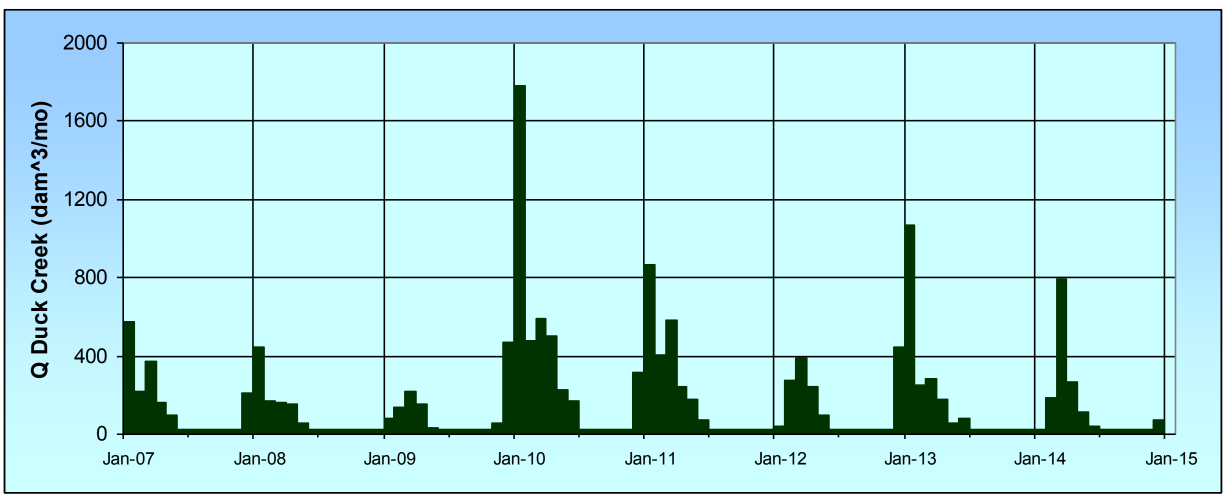


Figure 3.4 Duck Creek outflow in dam^3/mo.

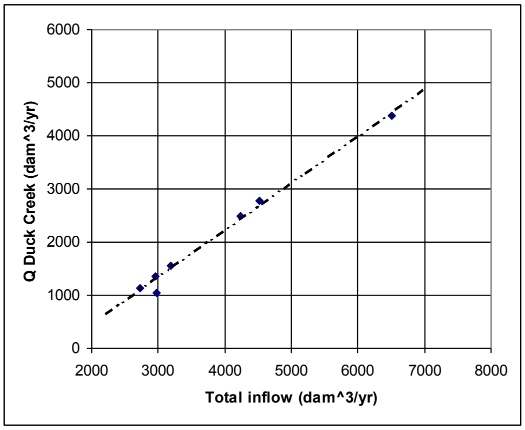


Figure 3.5 Q vs I.

Regression line Q = 089 I - 1329 R^2=0.99

Table 3.2 Comparison of Duck Creek outflow Q with total inflow I.

	Duck creek out Q	Total inflow I	Q/I
Winter of	dam^3/yr	dam^3/yr	
2007-08	1348	2958	46%
2008-09	797	2727	29%
2009-10	4368	6508	67%
2010-11	2779	4529	61%
2011-12	1211	2972	41%
2012-13	2493	4235	59%
2013-14	1565	3214	49%

3.4 Net Inflows

So far, these statistics are conventional. It is of interest to also examine the phasing of inflow and export of water; specifically, the inflows associated with storm water runoff and the export produced by Duck Creek outflow and withdrawals. In the notation defined previously, net inflow V is defined as:

$$V = (I - R) - (Q+W) \quad (6)$$

where R = direct rainfall to the lake surface and I is the solution to (3).

$$R = P \times A / 10^6 \ (\text{dam}^3) \quad (7)$$

where P = precipitation in mm and A is the lake surface area in m^2. If we let SW = (I - R), SW provides an estimate of the storm water runoff, defined as the amount of rainfall on the land portion of the watershed that flows into the lake. Then (6) becomes

$$V = SW - (Q+W) \quad (8)$$

(a) Variation of Total Inflow and Export

The monthly variation in I and (Q+W) is shown in Fig. 3.6. The difference between these two parameters, I-(Q+W), is illustrated in Fig. 3.7. Annual totals are listed in Table 3.3. These data show that, first, total inflow exceeds outflow, by an average of about 1,100 dam^3/yr (or (Q+W)/I ~ 69 ± 9%). This is expected, indeed necessary, since the inflows must balance the loss to evaporation and the replenishment of lake storage. Second, the data show that the majority of this imbalance occurs during the fall-early winter period (roughly from November to January). This coincides with the onset of fall rains during the period when the lake level is below the weir crest and the lake is filling. Also, withdrawals are reaching their annual minimums (i.e., Q+W is relatively low) at this time.

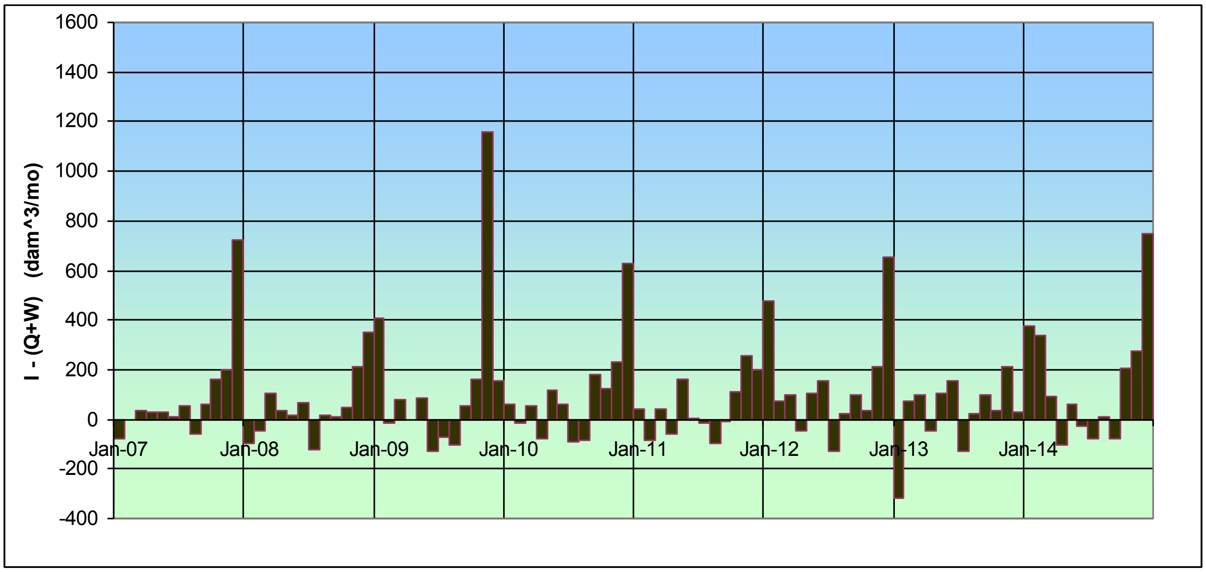


Figure 3.7 Monthly variation of I – (Q+W).

(b) Storm Water Inflows and the Importance of Direct Rainfall

The rainfall function R of equation (7) is shown in Fig. 3.8. The average annual rainfall volume on the lake surface is approximately 1,600 dam^3, which is only a little under one-half of the total inflow of about 3,800 dam^3. Thus, rainfall is an important contribution to the water balance.

The storm water runoff function SW = (I-R) is plotted in Fig. 3.9 (note the change in y-axis scale between Fig. 3.8 and 3.9). The annual average for SW is about 2,200 dam^3. The SW function provides an estimate of overland runoff, which is an important source of nutrients to the lake. In this case, it also contains and shallow ground water input.

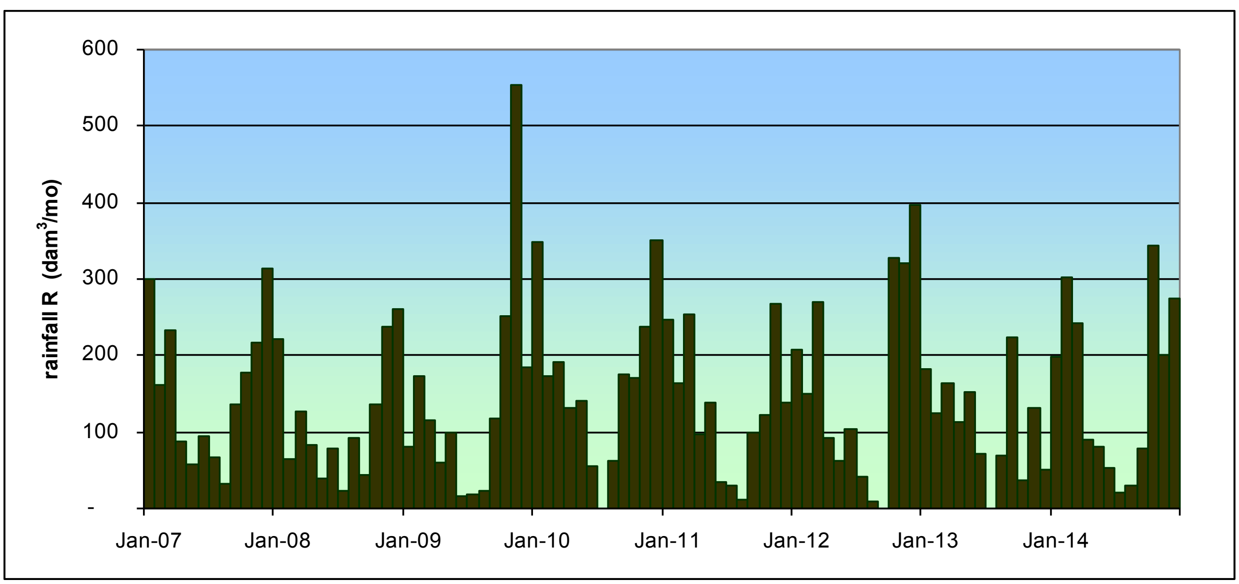


Figure 3.8 Rainfall function R in dam^3/mo.

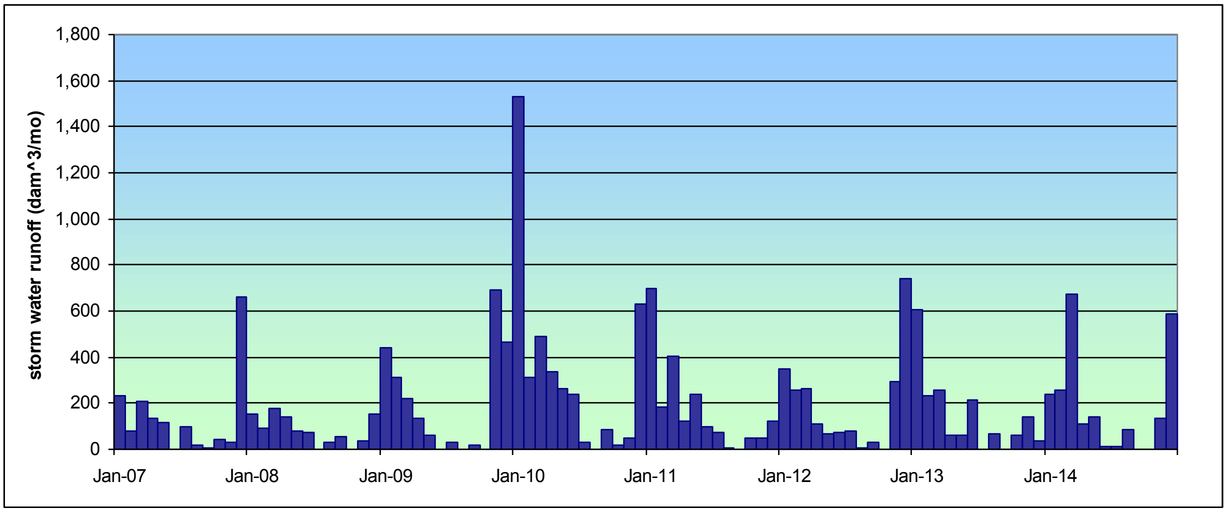


Figure 3.9 Storm water runoff function SW = (I - R) in dam^3/mo.

The correlation of runoff SW with total inflow I is shown in Fig. 3.10. The linear regression line is

$$SW = 0.79\,I - 826 \tag{9}$$

with $R^2 = 0.99$.

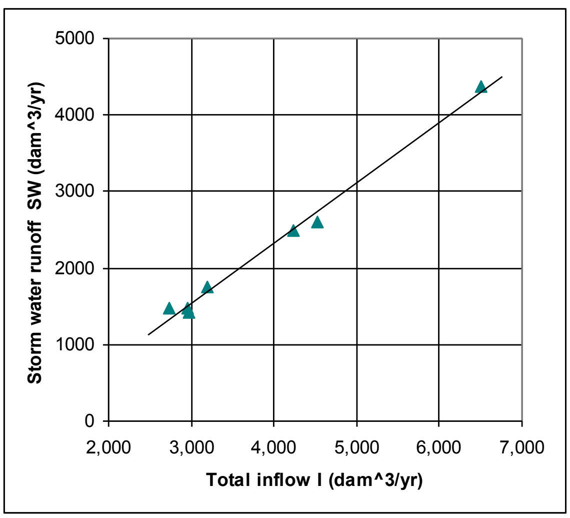


Figure 3.10 Runoff SW vs inflow I.

(c) Net Inflow

A comparison of storm water runoff SW and the total outflow from the lake (Q+W) is shown in Fig. 3.11. The export of water exceeds storm water runoff, by an average of 467 (208,763 n=7) dam^3/yr (Table 3.3 column 6). This difference averages about 17 ± 5%. The inter-annual variation in both terms is large, however, ranging well over a factor of two and is correlated with total precipitation.

The net inflow function V (equation (8)) is shown in Fig. 3.12 – this is another way of illustrating the difference between the bars in Fig. 3.11, broken down by month. The idea with this function is to show the relative magnitude of storm water runoff to the total export of water out Duck Creek and withdrawals, and the timing of the difference during the annual rainfall cycle, these being important external sources and sinks of nutrients to the lake.

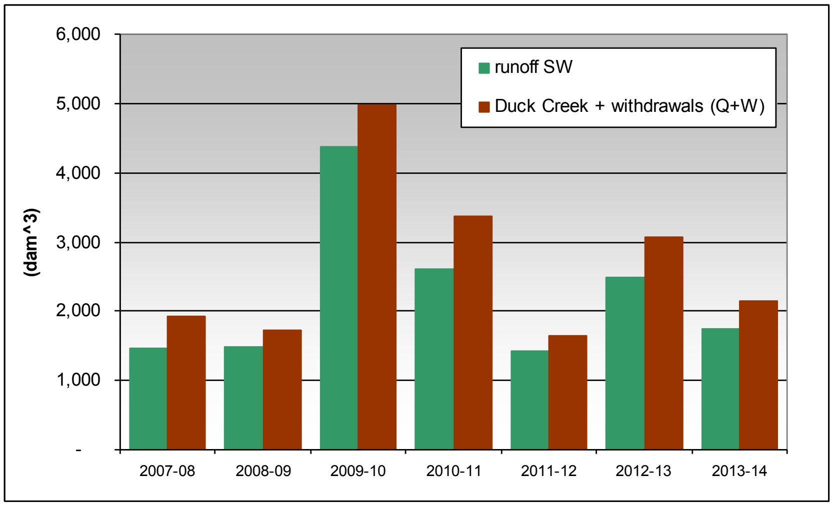


Figure 3.11 Comparison of runoff SW with total outflows (Q+W).

Typically, storm water runoff is greater than export for only 1 or 2 months each year, usually during the heaviest month(s) of fall-winter rains (November-January). These months, particularly November, tend to coincide with the time of year that nutrient loading in the lake through internal processes is highest.

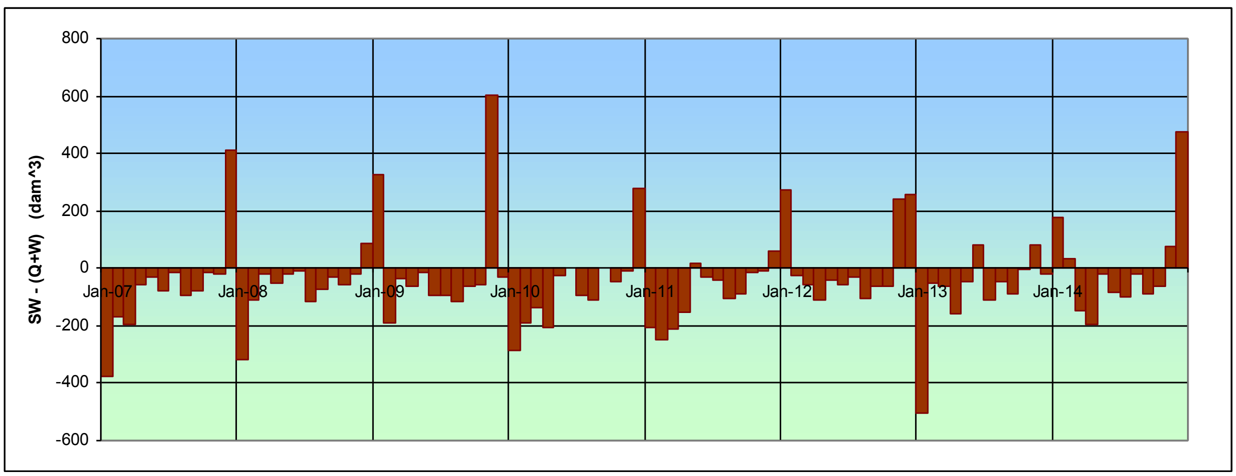


Figure 3.12 Net inflow function V (equation (8)) in dam^3/mo.

4.0 DYNAMICAL WATER BALANCE FUNCTION

In this chapter, an annualized static balance relation is set aside in favour of a dynamical water balance function. The objective is to illustrate the important seasonal variations in the sources and sinks of water, as well as the inter-annual variations. The results are derived from the monthly sums of the inflows and exports/losses of water, presented in absolute units (dam^3/mo) and as relative fractions expressed as percentages of the total of inflows and exports. Specifically, Figure 4.1 shows

R, SW, Q, E, W as a stacked bar graph,

and Fig. 4.2 shows

R:S, SW:S, Q:S, E:S, W:S as a stacked bar graph,

Where $S = |R| + |SW| + |Q| + |E| + |W|$[6] (10)

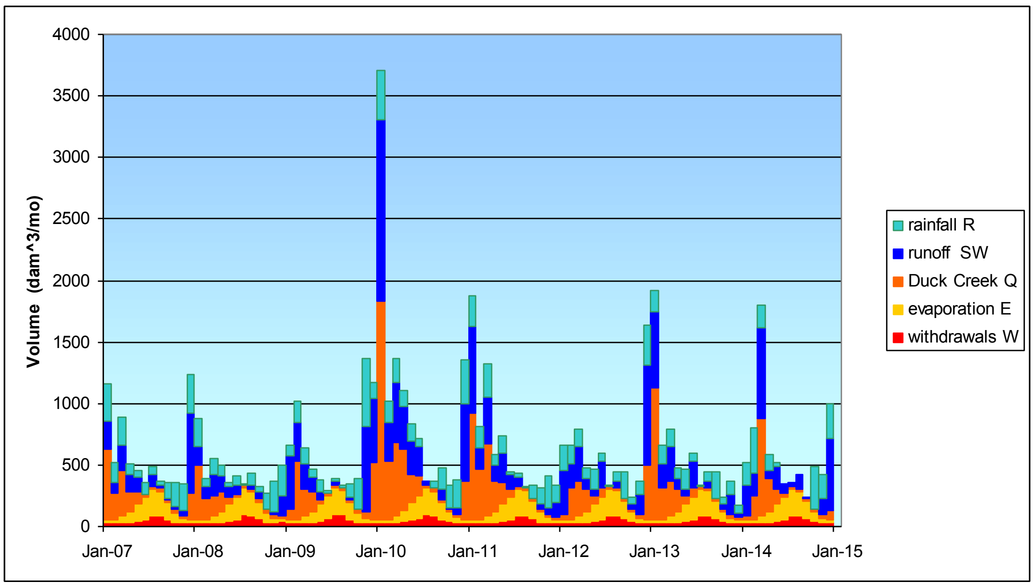


Figure 4.1 Monthly time-series of inflows and exports in dam^3/mo.

In Fig. 4.2 these data are portrayed as fractions of the annual total for S to show the monthly variation with respect to a full year. However, if the relative contribution of each term is computed as a fraction of the monthly total for S (Fig. 4.3) the variations during the year, and inter-annually are easier to see. Although in this case, the absolute changes in the total of the terms is lost.

There is a great deal of seasonal and inter-annual variation evident in Fig. 4.2 and 4.3; however, trends are present and are summarized in Table 4.1 in terms of averages over the eight years of data. At the onset of the rainy season in October and November, rainfall to the lake makes the greatest contribution of water, with less coming from runoff. This is logical since at that time of year the ground is dry and absorbs much of the early rainfall. Importantly, the lake is filling and there is little to no outflow to Duck Creek beyond the fish conservation flow. Similarly, evaporation is declining with cooler weather; together evaporation and withdrawals account for roughly 17% of the balance.

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[6] The | | symbol denotes absolute value of the variable.

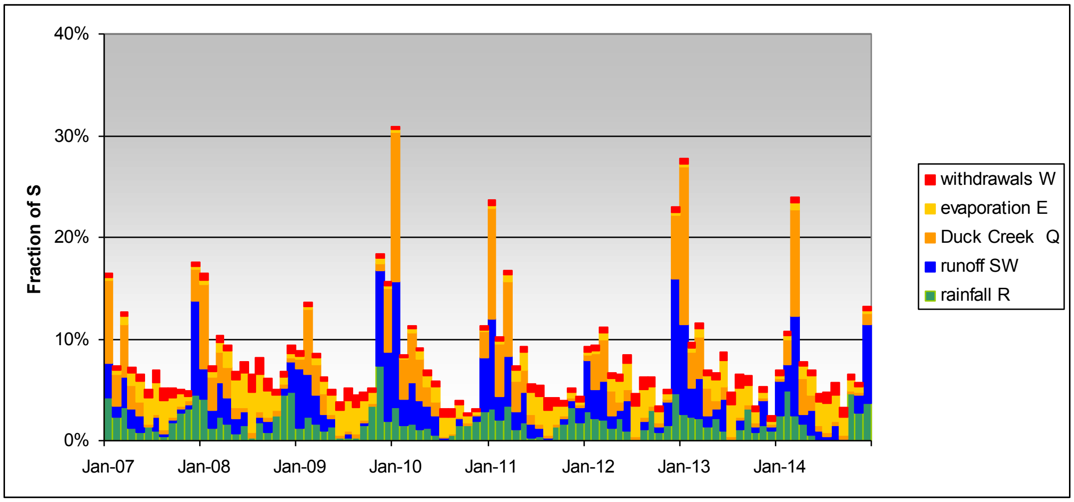

Figure 4.2 Actual proportions of inflows and exports as a fraction of the annual total of all inflows and exports. In this graph, the fractions sum to 100% over each 12-month period.

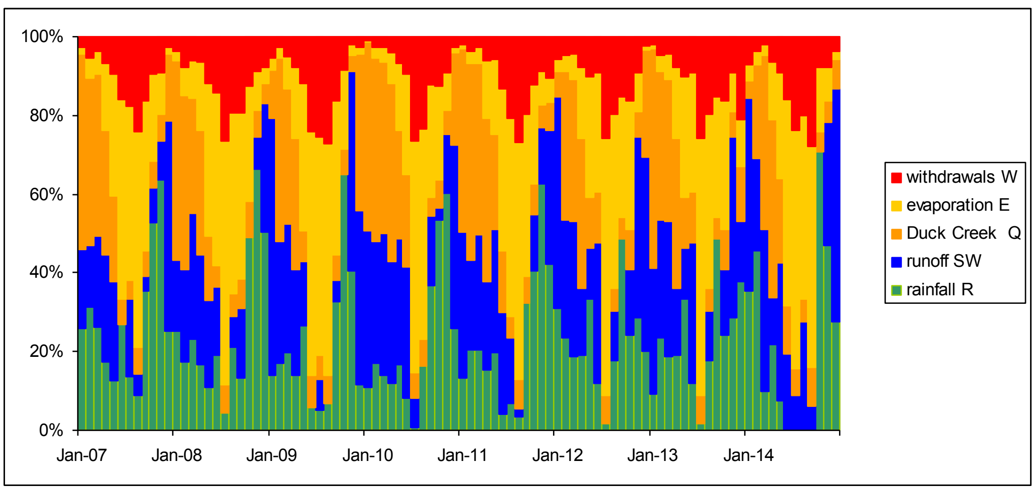

Figure 4.3 Relative proportions of inflows and exports as a fraction of the total of all inflows and exports calculated monthly. Here, the monthly totals equal 100% in order to show the relative importance of the different terms with time.

During winter, inflows shift in favour of runoff, by about 2:1 over rainfall on the lake, and Duck Creek outflow now accounts for about 25-30% of the balance. Over the spring months, outflows to Duck Creek usually continue (~38%), roughly balancing rainfall inflows (~39%), and the relative importance of evaporation and withdrawals increases (~20-25%).

The summer balance shows the complete reversal of the situation in winter when outflows have been reduced to the fish conservation flow, and there is little rainfall. At this time, evaporation plays the largest role (~50% or greater in July), withdrawals are about one-half of evaporation. Rainfall and outflows accounts for about 20% in total.

Between seasons, there exists a reasonably smooth transition between the percentages shown in this table. However, it is important to emphasize that while this seasonal breakdown is useful for a generalized description, the timing varies considerably from year to year. This follows from the observation that in some years, the wet season begins as early as October, while in others it can be delayed into late December (for example, the winters of 2008-09 and 2013-14 – Fig. 2.6 are "late" wet seasons).

Table 4.1 Relative water balance for different seasons of the year (averaged over eight years of data shown in Fig. 4.3). The proportions for each season total 100%.

	rainfall to lake	runoff	Duck Creek outflow	evaporation	withdrawals
onset of rainy season Oct-Nov	45%	30%	8%	10%	7%
winter wet season Dec-Feb	20%	40%	27%	6%	7%
spring-early summer Mar-June	17%	22%	38%	16%	7%
summer dry season Jul-Sep	4%	9%	8%	54%	25%

5.0 CONCLUSIONS

The water balance model provides a useful basis for defining some hydrological parameters for St. Mary Lake. Specifically, the total inflow can be related to the total precipitation measured beside the lake. From this parameter, quantitative estimates can then be made for the outflow to Duck Creek, and for inflow to the lake from storm water runoff. These last two parameters, together with water withdrawals, are essential for determining the relative importance of external sources and sinks of nutrients.

If we consider the balance between storm water runoff, and water removed from the lake, the calculations show that it is positive for only one or two months each year (storm water in exceeds flows out). These usually occur between November and January. In five of the eight years examined, the greatest amounts of runoff were found in November and December, when nutrient loading through internal processes is highest. In other months, outflows and withdrawals typically exceed runoff.

These results do not provide information on the nutrient balance itself, only the water balance. In a companion paper[7] the functions derived here for storm water runoff (SW) and water export (Q+W) are combined with measured values for phosphorus to provide estimates of that nutrient's inputs and exports.

ACKNOWLEDGEMENTS

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[7] Hodgins, D.O., 2015. Preliminary estimates of storm water phosphorus loads to St. Mary Lake, British Columbia. Submitted to the Technical Advisory Committee of the Salt Spring Island Watershed Protection Authority in September 2015.

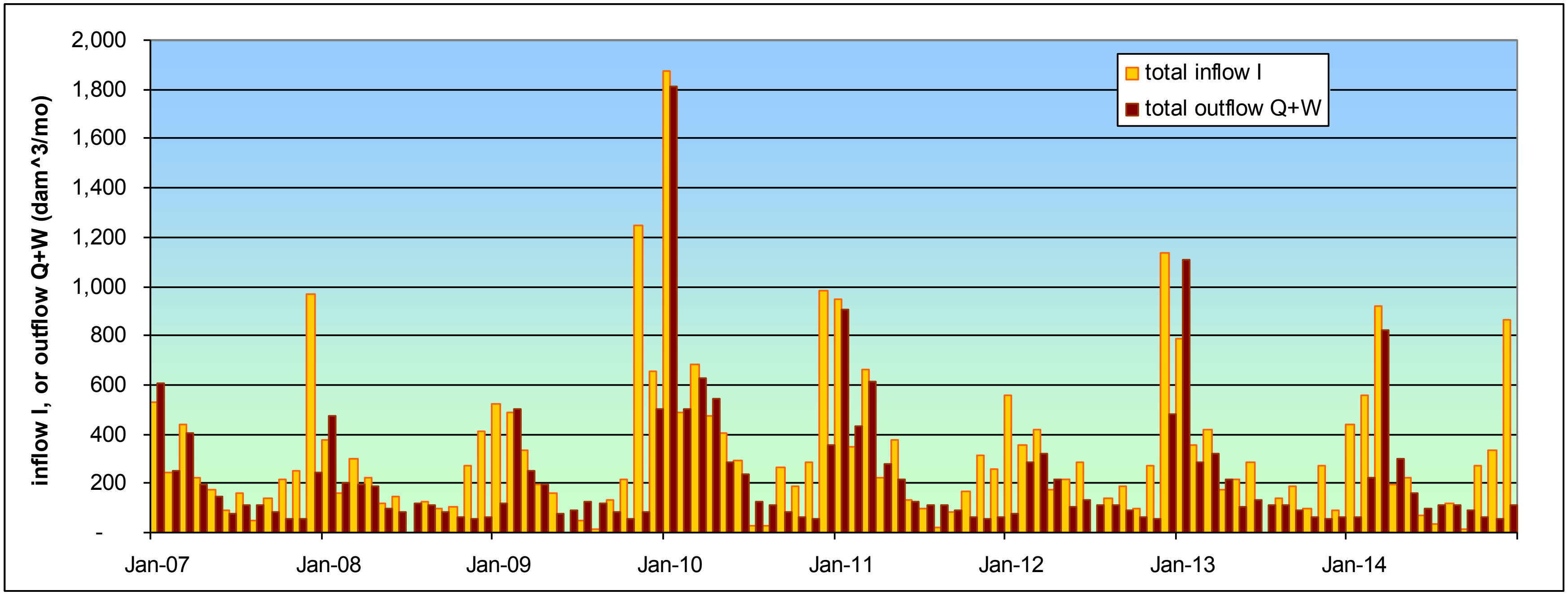

Figure 3.6 Monthly variation in total inflow I, and total outflow (Q+W).

Table 3.3 Annual totals for I, P, (Q+W, I-(Q+W), SW-(Q+W) and SW.

winter of	annual inflow I (dam^3)	annual Precip P (mm)	annual Q+W (dam^3)	annual I-(Q+W) (dam^3)	annual SW-(Q+W) (dam^3)	annual SW (dam^3)
2007-08	2958	832	1918	1040	-446	1473
2008-09	2727	733	1734	993	-258	1476
2009-10	6508	1194	4981	1527	-603	4378
2010-11	4529	1078	3366	1163	-763	2603
2011-12	2972	860	1641	1332	-215	1426
2012-13	4239	1024	3076	1163	-581	2495
2013-14	3198	824	2142	1055	-396	1747
mean	3876	935	2694	1182	-466	2228
min	2727	733	1641	993	-763	1426
max	6508	1194	4981	1527	-215	4378